Iowa State University

ECpE Department

EE653 Power distribution system modeling, optimization and simulation

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Modeling Shunt Admittance of Overhead and Underground Lines

Acknowledgement: The slides are developed based in part on Distribution System Modeling and Analysis, 4th edition, William H. Kersting, CRC Press, 2017

Overview

Conductor Types (materials) & Construction Info (spacings)

Self and Mutual Potential Coefficients

Primitive Potential Coefficient Matrix $[\hat{P}_{primitive}]$

Partition Form

$$[\hat{P}_{primitive}] = \begin{bmatrix} [\hat{P}_{ij}] & [\hat{P}_{in}] \\ [\hat{P}_{nj}] & [\hat{P}_{nn}] \end{bmatrix}$$

Kron reduction

Phase Potential Coefficient Matrix $[P_{abc}]$

Capacitance Matrix $[C_{abc}]$

Phase Shunt Admittance Matrix $[y_{abc}]$

Shunt Admittance of Overhead and Underground Lines

- The shunt admittance of a line consists of the conductance and the capacitive susceptance. The conductance is usually ignored because it is very small compared to the capacitive susceptance.
- The capacitance of a line is the result of the potential difference between conductors. A charged conductor creates an electric field that emanates outward from the center of the conductor. Lines of equipotential are created that are concentric to the charged conductor. This is illustrated in Fig.1.

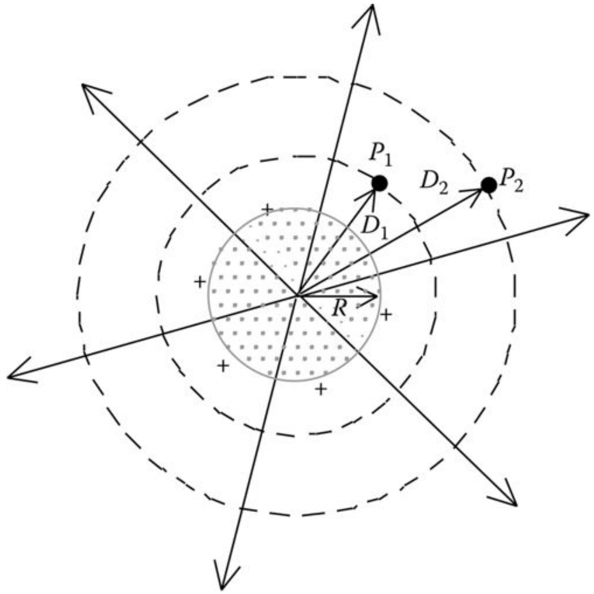


Fig.1 Electric field of a charged round conductor

Shunt Admittance of Overhead and Underground Lines

- In Fig.1, a difference of potential between two points (P_1 and P_2) is a result of the electric field of the charged conductor. When the potential difference between the two points is known, then the capacitance between the two points can be computed.
- If there are other charged conductors nearby, the potential difference between the two points will be a function of the distance to the other conductors and the charge on each conductor. The principle of superposition is used to compute the total voltage drop between two points and then the resulting capacitance between the points.
- The points can be points in space or the surface of two conductors or the surface of a conductor and ground.

General Voltage Drop Equation

- Fig.2 shows an array of N positively charged solid round conductors. Each conductor has a unique uniform charge density of q cb/m.
- The voltage drop between conductor i and conductor j as a result of all of the charged conductors is given by

$$V_{\mathrm{ij}}=\frac{1}{2\pi\varepsilon}\left(q_1 * \ln\frac{D_{1j}}{D_{1i}} + \cdots q_i * \ln\frac{D_{ij}}{RD_i} + \cdots q_j * \ln\frac{RD_j}{D_{ij}} + \cdots q_N * \ln\frac{D_{Nj}}{D_{Ni}}\right) \quad (1)$$

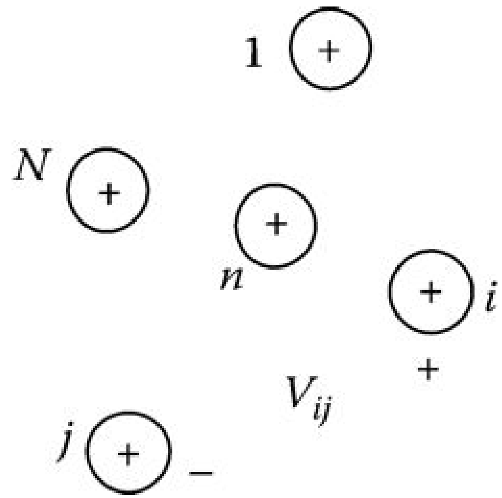


Fig.2 Array of round conductors

General Voltage Drop Equation

$$V_{\mathrm{ij}}=\frac{1}{2\pi\varepsilon}\ (q_1 * \ln\frac{D_{1j}}{D_{1i}} + \cdots q_i * \ln\frac{D_{ij}}{RD_i} + \cdots q_j * \ln\frac{RD_j}{D_{ij}} + \cdots q_N * \ln\frac{D_{Nj}}{D_{Ni}}) \tag{1}$$

Equation (1) can be written in a general form as

$$V_{\mathrm{ij}}=\frac{1}{2\pi\varepsilon}\sum_{n=1}^{N} q_N * \ln\frac{D_{nj}}{D_{ni}} \tag{2}$$

where

- $\varepsilon = \varepsilon_0\varepsilon_r$ is the permittivity of the medium, ε_0 is the permittivity of free space = 8.85×10^{-12} µF/m, ε_r is the relative permittivity of the medium
- q_n is the charge density on conductor n cb/m
- D_{ni} is the distance between conductor n and conductor i (ft)
- D_{nj} is the distance between conductor n and conductor j (ft)
- D_{nn} is the radius (RD_n) of conductor n (ft)

Overhead Lines

- The method of conductors and their images is employed in the calculation of the shunt capacitance of overhead lines.
- This is the same concept that was used in the general application of Carson's equations.
- Figure 3 illustrates the conductors and their images and will be used to develop a general voltage drop equation for overhead lines.

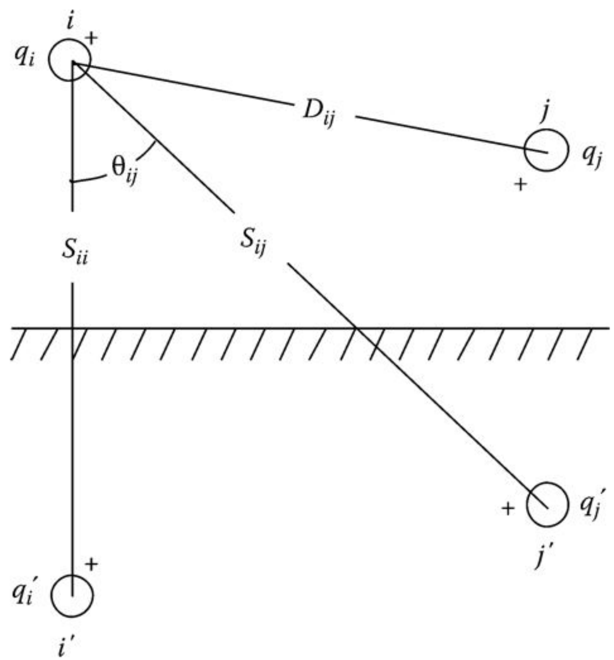


Fig.3 Conductors and images

Overhead Lines

In Fig.3 it is assumed that

$$q_i' = -q_i, q_j' = -q_j \quad (3)$$

Appling Equation (2) to Figure 5.3,

$$V_{\mathrm{ii}'} = \frac{1}{2\pi\varepsilon}\left(q_i * \ln\frac{S_{ii}}{RD_i} + q_i' * \ln\frac{RD_i}{S_{ii}} + q_j * \ln\frac{S_{ij}}{D_{ij}} + q_j' * \ln\frac{D_{ij}}{S_{ij}}\right) \quad (4)$$

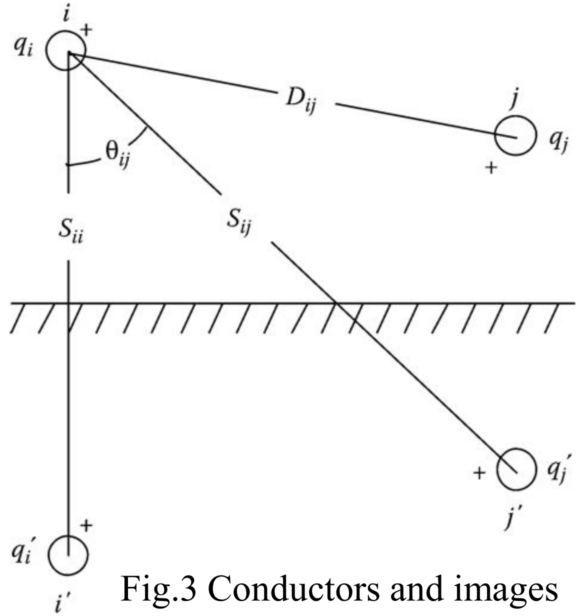


Fig.3 Conductors and images

Overhead Lines

$$q_i' = -q_i, q_j' = -q_j \quad (3)$$

$$V_{ii'} = \frac{1}{2\pi\varepsilon}\left(q_i * \ln\frac{S_{ii}}{RD_i} + q_i' * \ln\frac{RD_i}{S_{ii}} + q_j * \ln\frac{S_{ij}}{D_{ij}} + q_j' * \ln\frac{D_{ij}}{S_{ij}}\right) \quad (4)$$

Because of the assumptions of Equation (3), Equation (4) can be simplified to

$$V_{ii} = \frac{1}{2\pi\varepsilon}\left(q_i * \ln\frac{S_{ii}}{RD_i} - q_i * \ln\frac{RD_i}{S_{ii}} + q_j * \ln\frac{S_{ij}}{D_{ij}} - q_j * \ln\frac{D_{ij}}{S_{ij}}\right)$$

$$= \frac{1}{2\pi\varepsilon}\left(q_i * \ln\frac{S_{ii}}{RD_i} + q_i * \ln\frac{S_{ii}}{RD_i} + q_j * \ln\frac{S_{ij}}{D_{ij}} + q_j * \ln\frac{S_{ij}}{D_{ij}}\right)$$

$$= \frac{1}{2\pi\varepsilon}\left(2q_i * \ln\frac{S_{ii}}{RD_i} + 2q_j * \ln\frac{S_{ij}}{D_{ij}}\right) \quad (5)$$

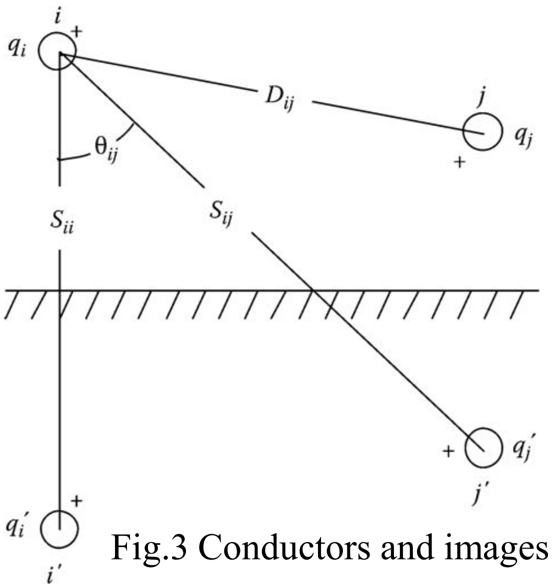


Fig.3 Conductors and images

where

- S_{ii} is the distance from conductor i to its image i' (ft)
- S_{ij} is the distance from conductor i to the image of conductor j' (ft)
- D_{ij} is the distance from conductor i to conductor j (ft)
- RD_i is the radius of conductor i in ft

Overhead Lines

$$V_{\mathrm{ii'}}=\frac{1}{2\pi\varepsilon}\left(2q_i * \ln\frac{S_{ii}}{RD_i} + 2q_j * \ln\frac{S_{ij}}{D_{ij}}\right) \qquad (5)$$

Equation (5) gives the total voltage drop between conductor *i* and its image. The voltage drop between conductor *i* and ground will be one-half of that given in Equation (5):

$$V_{\mathrm{ig}}=\frac{1}{2\pi\varepsilon}\left(q_i * \ln\frac{S_{ii}}{RD_i} + q_j * \ln\frac{S_{ij}}{D_{ij}}\right) \qquad (6)$$

Equation (6) can be written in general form as

$$V_{\mathrm{ig}}=\hat{P}_{ii} * q_i + \hat{P}_{ij} * q_j \qquad (7)$$

where $\hat{P}_{ii}$ and $\hat{P}_{ij}$ are the self- and mutual "potential coefficients."

For overhead lines the relative permittivity of air is assumed to be 1.0 so that

$$\varepsilon_0=1.0\times 8.85\times 10^{-12}\ F/m,\ \varepsilon_{\mathrm{air}}=1.4240\times 10^{-2}\ \mu F/mile \qquad (8)$$

Using the value of permittivity in μF/mile, the self- and mutual potential coefficients are defined as

$$\hat{P}_{ii} = 11.17689 * \ln\frac{S_{ii}}{RD_i}\ \mathrm{mile}/\mu F \qquad (9)$$

$$\hat{P}_{ij} = 11.17689 * \ln\frac{S_{ij}}{D_j}\ \mathrm{mile}/\mu F \qquad (10)$$

Overhead Lines

$$\hat{P}_{ii} = 11.17689 * \ln \frac{S_{ii}}{RD_i} \text{ mile}/\mu F \qquad (9)$$

$$\hat{P}_{ij} = 11.17689 * \ln \frac{S_{ij}}{D_j} \text{ mile}/\mu F \qquad (10)$$

Note: In applying Equations (9) and (10), the values of RD_i, S_{ii}, S_{ij}, and D_{ij} must all be in the same units. For overhead lines the distances between conductors are typically specified in feet while the value of the conductor diameter from a table will typically be in inches. Care must be taken to ensure that the radius in feet is used in applying the two equations.

For an overhead line of *ncond* conductors, the "primitive potential coefficient matrix" can be constructed. The primitive potential coefficient matrix will be an *ncond* × *ncond* matrix. For a four-wire grounded wye line the primitive coefficient matrix will be of the form

$$\left[\hat{P}_{primitive}\right] = \begin{bmatrix} \hat{P}_{aa} & \hat{P}_{ab} & \hat{P}_{ac} & \cdot & \hat{P}_{an} \\ \hat{P}_{ba} & \hat{P}_{bb} & \hat{P}_{bc} & \cdot & \hat{P}_{bn} \\ \hat{P}_{ca} & \hat{P}_{cb} & \hat{P}_{cc} & \cdot & \hat{P}_{cn} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \hat{P}_{na} & \hat{P}_{nb} & \hat{P}_{nc} & \cdot & \hat{P}_{nn} \end{bmatrix} \qquad (11)$$

Overhead Lines

$$[\hat{P}_{primitive}] = \begin{bmatrix} \hat{P}_{aa} \hat{P}_{ab} \hat{P}_{ac} \cdot \hat{P}_{an} \\ \hat{P}_{ba} \hat{P}_{bb} \hat{P}_{bc} \cdot \hat{P}_{bn} \\ \hat{P}_{ca} \hat{P}_{cb} \hat{P}_{cc} \cdot \hat{P}_{cn} \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \hat{P}_{na} \hat{P}_{nb} \hat{P}_{nc} \cdot \hat{P}_{nn} \end{bmatrix} \qquad (11)$$

The dots (·) in Equation (11) are partitioning the matrix between the third and fourth rows and columns. In partitioned form Equation (11) becomes

$$[\hat{P}_{primitive}] = \begin{bmatrix} [\hat{P}_{ij}] & [\hat{P}_{in}] \\ [\hat{P}_{nj}] & [\hat{P}_{nn}] \end{bmatrix} \qquad (12)$$

Because the neutral conductor is grounded, the matrix can be reduced using the "Kron reduction" method to an n-phase × n-phase phase potential coefficient matrix $[P_{abc}]$ given by

$$[P_{abc}] = [\hat{P}_{ij}] - [\hat{P}_{in}] * [\hat{P}_{nn}]^{-1} * [\hat{P}_{jn}] \qquad (13)$$

The inverse of the potential coefficient matrix will give the n-phase × n-phase capacitance matrix $[C_{abc}]$:

$$[C_{abc}] = [\hat{P}_{abc}]^{-1} \qquad (14)$$

Overhead Lines

$$[C_{abc}] = [P_{abc}]^{-1} \quad (14)$$

For a two-phase line the capacitance matrix of Equation (14) will be 2 × 2. A row and a column of zeros must be inserted for the missing phase. For a single-phase line, Equation (14) will result in a single element. Again rows and columns of zero must be inserted for the missing phase. In the case of the single-phase line, the only nonzero term will be that of the phase in use.

Neglecting the shunt conductance, the phase shunt admittance matrix is given by

$$[y_{abc}] = 0 + j * \omega * [C_{abc}] \ \mu S/mile \quad (15)$$

where

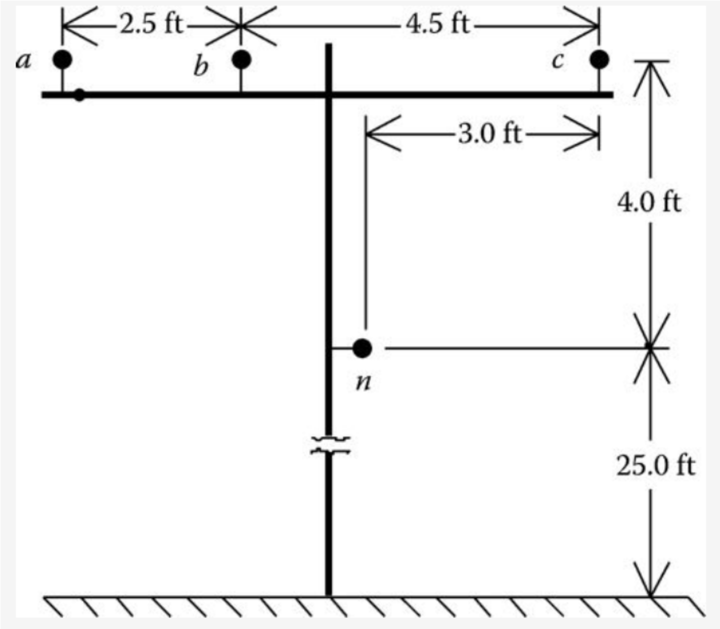
$$\omega = 2 * \pi * f = 376.9911$$

Example 1

Determine the shunt admittance matrix for the overhead line. Assume that the neutral conductor is 25 ft above ground.

The diameters of the phase and neutral conductors from the conductor table (Appendix A) are:

Conductor, 336,400 26/7 ACSR, $d_c = 0.721\ in., RD_c = 0.03004\ ft$; 4/0 6/1 ACSR, $d_c = 0.563\ in., RD_c = 0.02346\ ft$



Overhead line of Example 4.1

Example 1

Using the Cartesian coordinated in Example 4.1, the image distance matrix is given by

$$S_{ij} = \left| d_i - d_j^* \right|$$

Where d_j^* is the conjugate of d_j

For the configuration the distances between conductors and images in matrix form are

$$[S] = \begin{bmatrix} 58 & 58.0539 & 58.4209 & 54.1479 \\ 58.0539 & 58 & 58.1743 & 54.0208 \\ 58.4209 & 58.1743 & 58 & 54.0833 \\ 54.1479 & 54.0208 & 54.0833 & 50 \end{bmatrix} ft$$

The self-primitive potential coefficient for phase *a* and the mutual primitive potential coefficient between phases *a* and *b* are

$$\hat{P}_{aa} = 11.17689 * \ln \frac{58}{0.03004} = 84.5600 \text{ mile}/\mu F$$

$$\hat{P}_{ab} = 11.17689 * \ln \frac{58.0539}{2.5} = 35.1522 \text{ mile}/\mu F$$

Example 1

$$\hat{P}_{ii} = 11.17689 * \ln \frac{S_{ii}}{RD_i} \text{ mile}/\mu F \qquad (9)$$

$$\hat{P}_{ij} = 11.17689 * \ln \frac{S_{ij}}{D_j} \text{ mile}/\mu F \qquad (10)$$

Using Equations (9) and (10), the total primitive potential coefficient matrix is computed to be

$$[\hat{P}_{primitive}] = \begin{bmatrix} 84.5600 & 35.1522 & 23.7147 & 25.2469 \\ 35.1522 & 84.5600 & 28.6058 & 28.3590 \\ 23.7147 & 28.6058 & 84.5600 & 26.6131 \\ 25.2469 & 28.3590 & 26.6131 & 85.6659 \end{bmatrix} \text{mile}/\mu F$$

Since the fourth conductor (neutral) is grounded, the Kron reduction method is used to compute the “phase potential coefficient matrix.” Because only one row and one column need to be eliminated, the $[\hat{P}_{nn}]$ term is a single element so that the Kron reduction equation for this case can be modified to

$$P_{ij} = \hat{P}_{ij} - \frac{\hat{P}_{i4} * \hat{P}_{j4}}{\hat{P}_{44}}$$

where $i = 1, 2, 3$ and $j = 1, 2, 3$.

Example 1

For example, the value of P_{cb} is computed to be

$$P_{cb} = \hat{P}_{32} - \frac{\hat{P}_{34} * \hat{P}_{24}}{\hat{P}_{44}} = 28.6058 - \frac{26.6134 * 28.359}{85.6659} = 19.7957$$

Following the Kron reduction, the phase potential coefficient matrix is

$$[P_{abc}] = \begin{bmatrix} 77.1194 & 26.7944 & 15.8714 \\ 26.7944 & 75.1720 & 19.7957 \\ 15.8714 & 19.7957 & 76.2923 \end{bmatrix}$$

Invert $[P_{abc}]$ to determine the shunt capacitance matrix:

$$[C_{abc}] = [P]^{-1} = \begin{bmatrix} 0.015 & -0.0049 & -0.0019 \\ -0.0049 & 0.0159 & -0.0031 \\ -0.0019 & -0.0031 & 0.0143 \end{bmatrix}$$

Multiply $[C_{abc}]$ by the radian frequency to determine the final three-phase shunt admittance matrix:

$$[y_{abc}] = j*376.9911*[C_{abc}] = \begin{bmatrix} j5.6711 & -j1.8362 & -j0.7033 \\ -j1.8362 & j5.9774 & -j1.1690 \\ -j0.7033 & -j1.169 & j5.3911 \end{bmatrix} \mu S/mile$$

Concentric Neutral Cable Underground Lines

Most underground distribution lines consist of one or more concentric neutral cables. Fig.4 illustrates a basic concentric neutral cable with center conductor being the phase conductor and the concentric neutral strands displaced equally around a circle of radius R_b.

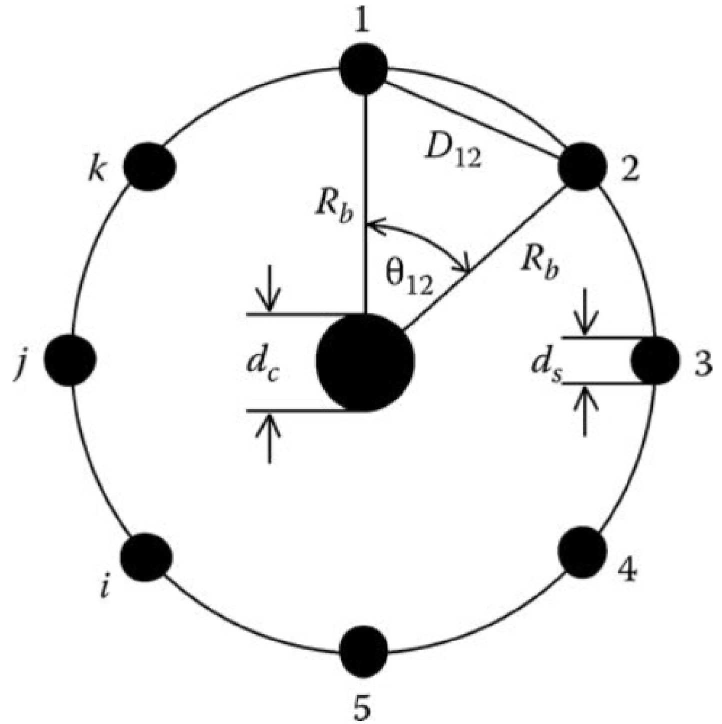


Fig.4 Conductors and images

Concentric Neutral Cable Underground Lines

Referring to Fig.4 the following definitions apply:

R_b represents the radius of a circle passing through the centers of the neutral strands.

d_c represents the diameter of the phase conductor.

d_s represents the diameter of a neutral strand.

k represents the total number of neutral strands.

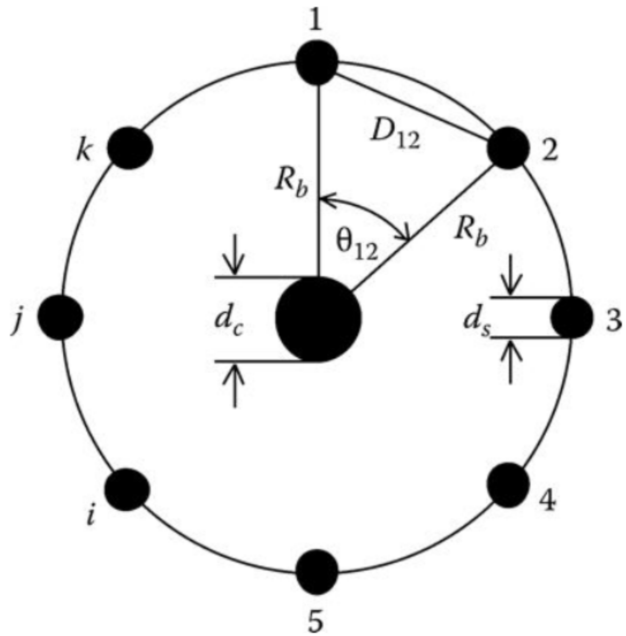


Fig.4 Conductors and images

Concentric Neutral Cable Underground Lines

$$V_{\text{ij}}=\frac{1}{2\pi\varepsilon}\sum_{n=1}^{N} q_N * \ln\frac{D_{nj}}{D_{ni}} \tag{2}$$

The concentric neutral strands are grounded so that they are all at the same potential. Because of the stranding, it is assumed that the electric field created by the charge on the phase conductor will be confined to the boundary of the concentric neutral strands. In order to compute the capacitance between the phase conductor and ground, the general voltage drop of Equation (2) will be applied. Since all of the neutral strands are at the same potential, it is necessary to determine only the potential difference between the phase conductor p and strand 1.

$$V_{p1}=\frac{1}{2\pi\varepsilon}\left(q_p * \ln\frac{R_b}{RD_c} + q_1 * \ln\frac{RD_s}{R_b} + q_2 * \ln\frac{D_{12}}{R_b} + \cdots q_i * \ln\frac{D_{1i}}{R_b} + \cdots q_k * \ln\frac{D_{k1}}{R_b}\right)$$

where (16)

$$RD_c = \frac{d_c}{2}, RD_s = \frac{d_s}{2}$$

It is assumed that each of the neutral strands carries the same charge such that

$$q_1 = q_2 = q_i = q_k = -\frac{q_p}{k} \tag{17}$$

Concentric Neutral Cable Underground Lines

$$V_{p1} = \frac{1}{2\pi\varepsilon}\ \left(q_p * \ln\frac{R_b}{RD_c} + q_1 * \ln\frac{RD_s}{R_b} + q_2 * \ln\frac{D_{12}}{R_b} + \cdots q_i * \ln\frac{D_{1i}}{R_b} + \cdots q_k * \ln\frac{D_{k1}}{R_b}\right) \quad (16)$$

Equation (16) can be simplified to

$$V_{p1} = \frac{1}{2\pi\varepsilon}\left[q_p * \ln\frac{R_b}{RD_c} - \frac{q_p}{k}\left(\ln\frac{RD_s}{R_b} + \ln\frac{D_{12}}{R_b} + \cdots + \ln\frac{D_{1i}}{R_b} + \cdots + \ln\frac{D_{k1}}{R_b}\right)\right]$$

$$= \frac{q_p}{2\pi\varepsilon}\left[\ln\frac{R_b}{RD_c} - \frac{1}{k}\left(\ln\frac{RD_s * D_{12} * D_{1i} \ldots D_{1k}}{R_b^k}\right)\right] \quad (18)$$

Concentric Neutral Cable Underground Lines

The numerator of the second ln term in Equation (18) needs to be expanded. The numerator represents the product of the radius and the distances between strand i and all of the other strands. Referring to Fig.4, the following relations apply:

$$\theta_{12} = \frac{2\pi}{k}$$

$$\theta_{13} = 2\theta_{12} = \frac{4\pi}{k}$$

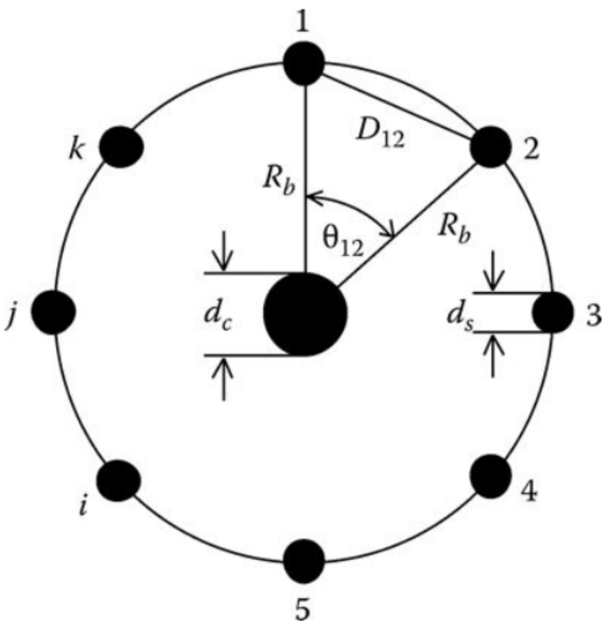


Fig.4 Conductors and images

In general, the angle between strand #1 and any other strand #i is given by

$$\theta_{1i} = (i-1)\theta_{12} = \frac{(i-1) * 2\pi}{k}$$

Concentric Neutral Cable Underground Lines

The distances between the various strands are given by

$$D_{12} = 2 * R_b * \sin(\frac{\theta_{12}}{2}) = 2 * R_b * \sin(\frac{\pi}{k})$$
$$D_{13} = 2 * R_b * \sin(\frac{\theta_{13}}{2}) = 2 * R_b * \sin(\frac{2\pi}{k}) \quad (20)$$

The distance between strand 1 and any other strand *i* is given by

$$D_{1i} = 2 * R_b * \sin(\frac{\theta_{1i}}{2}) = 2 * R_b * \sin(\frac{(i-1) * \pi}{k}) \quad (21)$$

Equation (21) can be used to expand the numerator of the second log term of Equation (18):

$$RD_s * D_{12} \dots D_{1i} \dots D_{1k}$$
$$= RD_s * R_b^{k-1}[2\sin\left(\frac{\pi}{k}\right) * 2\sin\left(\frac{2\pi}{k}\right) \dots 2\sin(\frac{(i-1)\pi}{k}) \dots 2\sin(\frac{(k-1)}{k})] \quad (22)$$

Concentric Neutral Cable Underground Lines

$$RD_s * D_{12} \dots D_{1i} \dots D_{1k}$$
$$= RD_s * R_b^{k-1}[2\sin\left(\frac{\pi}{k}\right) * 2\sin\left(\frac{2\pi}{k}\right) \dots 2\sin(\frac{(i-1)\pi}{k}) \dots 2\sin(\frac{(k-1)}{k})] \qquad (22)$$

$$[V_{LG}] = [P] * [q] \qquad (23)$$

The distance between strand 1 and any other strand *i* is given by

$$D_{1i} = 2 * R_b * \sin(\frac{\theta_{1i}}{2}) = 2 * R_b * \sin(\frac{(i-1) * \pi}{k}) \qquad (21)$$

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Concentric Neutral Cable Underground Lines

$$V_{p1}=\frac{q_p}{2\pi\varepsilon}\left[\ln\frac{R_b}{RD_c}-\frac{1}{k}\left(\ln\frac{\mathrm{k}*RD_s*R_b^{k-1}}{R_b^k}\right)\right]=\frac{q_p}{2\pi\varepsilon}\left[\ln\frac{R_b}{RD_c}-\frac{1}{k}\left(\ln\frac{\mathrm{k}*RD_s}{R_b}\right)\right] \qquad (24)$$

Since the neutral strands are all grounded, Equation (24) gives the voltage drop between the phase conductor and ground. Therefore, the capacitance from phase to ground for a concentric neutral cable is given by

$$C_{pg}=\frac{q_p}{V_{p1}}=\frac{2\pi\varepsilon}{\ln(R_b/RD_c)-(1/k)\ln(k*RD_s/R_b)} \qquad (25)$$

where

- $\varepsilon = \varepsilon_0\varepsilon_r$ is the permittivity of the medium
- ε_0 is the permittivity of free space = 0.01420 μF/mile
- ε_r is the relative permittivity of the medium

Concentric Neutral Cable Underground Lines

The electric field of a cable is confined to the insulation material. Various types of insulation material are used, and each will have a range of values for the relative permittivity. Table.1 gives the range of values of relative permittivity for four common insulation materials.

Table.1 Typical Values of Relative Permittivity (ϵ_r)

Material	Range of Value of Relative Permittivity
Polyvinyl chloride (PVC)	3.4-8.0
Ethylene-propylene rubber (EPR)	2.5-3.5
Polyethylene (PE)	2.5-2.6
Cross-linked polyethylene (XLPE)	2.3-6.0

Concentric Neutral Cable Underground Lines

Cross-linked polyethylene is a very popular insulation material. If the minimum value of relative permittivity is assumed as 2.3, the equation for the shunt admittance of the concentric neutral cable is given by

$$y_{ag} = 0 + j\frac{77.3619}{\ln(R_b/RD_c) - (1/k)\ln(k * RD_s/R_b)}\ \mu S/mile \tag{26}$$

Example 3

Determine the three-phase shunt admittance matrix for the concentric neutral line.

$$R_b = R = 0.0511\ ft = 0.631\ in$$

Diameter of the 250,000 AA phase conductor = 0.567 in. Therefore,

$$RD_c = \frac{0.567}{2} = 0.2835\ in$$

Diameter of the #14 CU concentric neutral strand = 0.0641 in. Therefore,

$$RD_s = \frac{0.0641}{2} = 0.03205\ in$$

Example 3

Substitute into (26)

$$y_{ag} = j\frac{77.3619}{\ln(R_b/RD_c) - (1/k)\ln(k * RD_s/R_b)} \tag{26}$$

$$y_{ag} = j\frac{77.3619}{\ln(0.6132/0.2835) - (1/13)\ln(13 * 0.03205/0.6132)}$$
$$= j96.6098\ \mu S/mile$$

The phase admittance for this three-phase underground line is

$$[y_{abc}] = \begin{bmatrix} j96.6098 & 0 & 0 \\ 0 & j96.6098 & 0 \\ 0 & 0 & j96.6098 \end{bmatrix} \mu S/mile$$

Tape-Shielded Cable Underground Lines

A tape-shielded cable is shown in Figure 5. Referring to Figure 5 R_b is the radius of a circle passing through the center of the tape shield. As with the concentric neutral cable, the electric field is confined to the insulation so that the relative permittivity of Table 1 will apply.

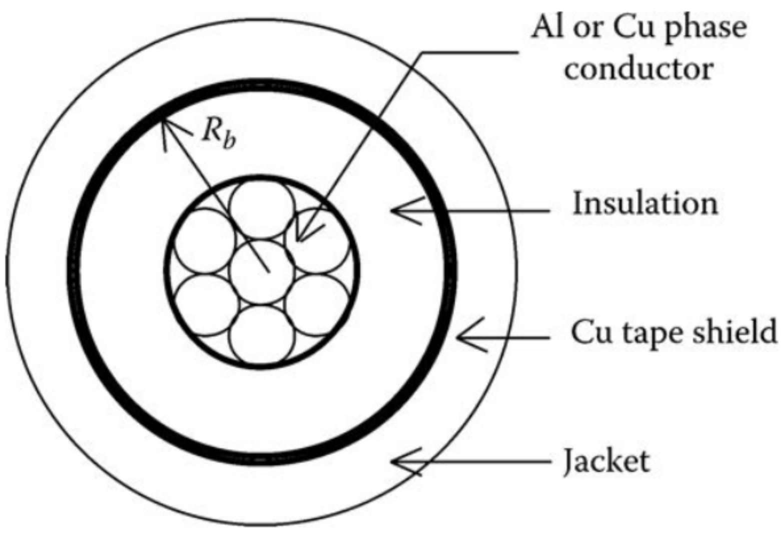

Fig.5 Tape-shielded conductor

Table 1 Typical Values of Relative Permittivity (ϵ_r)

Material	Range of Value of Relative Permittivity
Polyvinyl chloride (PVC)	3.4-8.0
Ethylene-propylene rubber (EPR)	2.5-3.5
Polyethylene (PE)	2.5-2.6
Cross-linked polyethylene (XLPE)	2.3-6.0

Tape-Shielded Cable Underground Lines

$$V_{p1} = \frac{1}{2\pi\varepsilon}\left[q_p * \ln\frac{R_b}{RD_c} - \frac{q_p}{k}\left(\ln\frac{RD_s}{R_b} + \ln\frac{D_{12}}{R_b} + \cdots + \ln\frac{D_{1i}}{R_b} + \cdots + \ln\frac{D_{k1}}{R_b}\right)\right]$$
$$= \frac{q_p}{2\pi\varepsilon}\left[\ln\frac{R_b}{RD_c} - \frac{1}{k}\left(\ln\frac{RD_s * D_{12} * D_{1i} \ldots D_{1k}}{R_b^k}\right)\right] \quad (18)$$

The tape-shielded conductor can be visualized as a concentric neutral cable where the number of strands *k* has become infinite. When *k* in Equation (18) approaches infinity, the second term in the denominator approaches zero. Therefore, the equation for the shunt admittance of a tape-shielded conductor becomes

$$y_{ag} = 0 + j\frac{77.3619}{\ln(R_b/RD_c)}\ \mu S/mile \quad (27)$$

Example 4

Determine the shunt admittance of the single-phase tape-shielded cable. Outside diameter of the tape shield is 0.88 in. The thickness of the tape shield (T) is 5 mil. The radius of a circle passing through the center of the tape shield is given by

$$T = \frac{5}{1000} = 0.005$$

$$R_b = \frac{d_s - T}{2} = \frac{0.88 - 0.005}{2} = 0.4375 \text{ in}$$

The diameter of the 1/0 AA phase conductor is 0.368 in. Therefore,

$$RD_c = \frac{d_p}{2} = \frac{0.368}{2} = 0.1840 \text{ in}$$

Substitute into Equation (27):

$$y_{ag} = j\frac{77.3619}{\ln(R_b/RD_c)} = j\frac{77.3619}{\ln(0.4375/0.184)} = j89.3179\ \mu S/mile$$

Example 5.4

The phase admittance for this three-phase underground line is

$$[y_{abc}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & j89.3179 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mu S/mile$$

Sequence Admittance

The sequence admittances of a three-phase line can be determined in much the same manner as the sequence impedances were determined. Assume that the 3 × 3 admittance matrix is given in S/mile. Then the three-phase capacitance currents as a function of the line-to-ground voltages are given by

$$\begin{bmatrix} Icap_a \\ Icap_b \\ Icap_c \end{bmatrix} = \begin{bmatrix} y_{aa} & y_{ab} & y_{ac} \\ y_{ba} & y_{bb} & y_{bc} \\ y_{ca} & y_{cb} & y_{cc} \end{bmatrix} \begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} \tag{28}$$

$$[Icap_{abc}] = [y_{abc}][VLG_{abc}] \tag{29}$$

Applying the symmetrical component transformations

$$[Icap_{012}] = [A_S]^{-1}[Icap_{abc}] = [A_S]^{-1}[y_{abc}][A_s][VLG_{012}] \tag{30}$$

Sequence Admittance

$$[Icap_{012}]=[A_S]^{-1}[Icap_{abc}]=[A_S]^{-1}[y_{abc}][A_s][VLG_{012}] \qquad (30)$$

From Equation (30), the sequence admittance matrix is given by

$$[\mathrm{y}_{012}] = [A_S]^{-1}[y_{abc}][A_s]=\begin{bmatrix} y_{00} & y_{01} & y_{02} \\ y_{10} & y_{11} & y_{12} \\ y_{20} & y_{21} & y_{22} \end{bmatrix} \qquad (31)$$

For a three-phase overhead line with unsymmetrical spacing, the sequence admittance matrix will be full. That is, the off-diagonal terms will be nonzero. However, a three-phase underground line with three identical cables will only have the diagonal terms since there is no "mutual capacitance" between phases. In fact, the sequence admittances will be exactly the same as the phase admittances.

Thank You!